Guidelines on the Cryptographic Algorithms to Accompany the Usage of Standards GOST R 34.10-2012 and GOST R 34.11-2012

Abstract

The purpose of this document is to make the specifications of the cryptographic algorithms defined by the Russian national standards GOST R 34.10-2012 and GOST R 34.11-2012 available to the Internet community for their implementation in the cryptographic protocols based on the accompanying algorithms.

These specifications define the pseudorandom functions, the key agreement algorithm based on the Diffie-Hellman algorithm and a hash function, the parameters of elliptic curves, the key derivation functions, and the key export functions.

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1. Introduction

The accompanying algorithms are intended for the implementation of cryptographic protocols. This memo contains a description of the accompanying algorithms based on the Russian national standards GOST R 34.10-2012 [GOST3410-2012] and GOST R 34.11-2012 [GOST3411-2012]. The English versions of these standards can be found in [RFC7091] and [RFC6986]; the English version of the encryption standard GOST 28147-89 [GOST28147-89] (which is used in the key export functions) can be found in [RFC5830].

The specifications of algorithms and parameters proposed in this memo are provided on the basis of experience in the development of the cryptographic protocols, as described in [RFC4357], [RFC4490], and [RFC4491].

This memo describes the pseudorandom functions, the key agreement algorithm based on the Diffie-Hellman algorithm and a hash function, the parameters of elliptic curves, the key derivation functions, and the key export functions necessary to ensure interoperability of security protocols that make use of the Russian cryptographic standards GOST R 34.10-2012 [GOST3410-2012] digital signature algorithm and GOST R 34.11-2012 [GOST3411-2012] cryptographic hash function.

2. Conventions Used in This Document

The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT", "SHOULD", "SHOULD NOT", "RECOMMENDED", "MAY", and "OPTIONAL" in this document are to be interpreted as described in [RFC2119].

3. Basic Terms, Definitions, and Notations

This document uses the following terms and definitions for the sets and operations on the elements of these sets:

(xor) Exclusive-or of two binary vectors of the same length.

V_n The finite vector space over GF(2) of dimension n, n >= 0, with the (xor) operation. For n = 0, the V_0 space consists of a single empty element of size 0. If U is an element of V_n, then U = (u_(n-1), u_(n-2), ..., u_1, u_0), where u_i in {0, 1}.
V_(8, r)  
The set of byte vectors of size r, r >= 0, for r = 0 the V_(8, r) set consists of a single empty element of size 0.
If W is an element of V_(8, r), r > 0, then W = (w^0, w^1, ..., w^(r-1)), where w^0, w^1, ..., w^(r-1) are elements of V_8.

Bit representation  
The bit representation of the element W = (w^0, w^1, ..., w^(r-1)) of V_(8, r) is an element (w_(8r-1), w_(8r-2), ..., w_1, w_0) of V_(8*r), where w^0 = (w_7, w_6, ..., w_0), w^1 = (w_15, w_14, ..., w_8), ..., w^(r-1) = (w_(8r-1), w_(8r-2), ..., w_(8r-8)) are elements of V_8.

Byte representation  
If n is a multiple of 8, r = n/8, then the byte representation of the element W = (w_(n-1), w_(n-2), ..., w_0) of V_n is a byte vector (w^0, w^1, ..., w^(r-1)) of V_(8, r), where w^0 = (w_7, w_6, ..., w_0), w^1 = (w_15, w_14, ..., w_8), ..., w^(r-1) = (w_(8r-1), w_(8r-2), ..., w_(8r-8)) are elements of V_8.

A|B  
Concatenation of byte vectors A and B, i.e., if A in V_(8, r1), B in V_(8, r2), A = (a^0, a^1, ..., a^(r1-1)) and B = (b^0, b^1, ..., b^(r2-1)), then A|B = (a^0, a^1, ..., a^(r1-1), b^0, b^1, ..., b^(r2-1)) is an element of V_(8, r1+r2).

K (key)  
An arbitrary element of V_n. If K in V_n, then its size (in bits) is equal to n, where n can be an arbitrary natural number.
This memo uses the following abbreviations and symbols:

+---------+------------------------------------------------------------------+
| Symbols | Meaning                                                          |
+---------+------------------------------------------------------------------+
| H_256   | GOST R 34.11-2012 hash function with 256-bit output              |
| H_512   | GOST R 34.11-2012 hash function with 512-bit output              |
| HMAC    | Hashed-based Message Authentication Code. A function for calculating a message authentication code, based on a hash function in accordance with [RFC2104] |
| PRF     | A pseudorandom function, i.e., a transformation that allows generation of a pseudorandom sequence of bytes |
| KDF     | A key derivation function, i.e., a transformation that allows keys and keying material to be derived from the root key and additional input using a pseudorandom function |
| VKO     | A key agreement algorithm based on the Diffie-Hellman algorithm and a hash function |

To generate a byte sequence of the size r with functions that give a longer output, the output is truncated to the first r bytes. This remark applies to the following functions:

- the functions described in Section 4.2;
- KDF_TREE_GOSTR3411_2012_256 described in Section 4.4;
- KDF_GOSTR3411_2012_256 described in Section 4.5.

Hereinafter, all data are provided in byte representation unless otherwise specified.

If a function is defined outside this document (e.g., H_256) and its definition requires arguments in bit representation, it is assumed that the bit representations of the arguments are formed immediately before the calculation of the function (in particular, immediately after the application of the operation (|) to the byte representation of the arguments).

If the output of another function defined outside of this document is used as an argument of the functions defined below and it has the bit representation, then it is assumed that an output MUST have a length
that is a multiple of 8 and that it will be translated into the byte
representation in advance.

When a point on an elliptic curve is given to an input of a hash
function, affine coordinates for short Weierstrass form are used (see
Section 5): an x coordinate value is fed first, a y coordinate value
is fed second, both in little-endian format.

4. Algorithm Descriptions

4.1. HMAC Functions

This section defines the HMAC transformations based on the GOST R
34.11-2012 [GOST3411-2012] algorithm.

4.1.1. HMAC_GOSTR3411_2012_256

This HMAC transformation is based on the GOST R 34.11-2012
[GOST3411-2012] hash function with 256-bit output. The object
identifier of this transformation is shown below:

```
id-tc26-hmac-gost-3411-12-256::= {iso(1) member-body(2) ru(643)
    rostandart(7) tc26(1) algorithms(1) mac(4) hmac-gost-
    3411-12-256(1)}.
```

This algorithm uses H_256 as a hash function for HMAC, described in
[RFC2104]. The method of forming the values of ipad and opad is also
specified in [RFC2104]. The size of HMAC_GOSTR3411_2012_256 output
is equal to 32 bytes, the block size of the iterative procedure for
the H_256 compression function is equal to 64 bytes (in the notation
of [RFC2104], L = 32 and B = 64, respectively).

4.1.2. HMAC_GOSTR3411_2012_512

This HMAC transformation is based on the GOST R 34.11-2012
[GOST3411-2012] hash function with 512-bit output. The object
identifier of this transformation is shown below:

```
id-tc26-hmac-gost-3411-12-512::= {iso(1) member-body(2) ru(643)
    rostandart(7) tc26(1) algorithms(1) mac(4) hmac-gost-
    3411-12-512(2)}.
```

This algorithm uses H_512 as a hash function for HMAC, described in
[RFC2104]. The method of forming the values of ipad and opad is also
specified in [RFC2104]. The size of HMAC_GOSTR3411_2012_512 output
is equal to 64 bytes, the block size of the iterative procedure for
the H_512 compression function is equal to 64 bytes (in the notation
of [RFC2104], L = 64 and B = 64, respectively).
4.2. Pseudorandom Functions

This section defines four HMAC-based PRF transformations recommended for usage. Two of them are designed for the Transport Layer Security (TLS) protocol and two are designed for the IPsec protocol.

4.2.1. PRFs for the TLS Protocol

4.2.1.1. PRF_TLS_GOSTR3411_2012_256

This is the transformation providing the pseudorandom function for the TLS protocol (1.0 and higher versions) in accordance with GOST R 34.11-2012 [GOST3411-2012]. It uses the P_GOSTR3411_2012_256 function that is similar to the P_hash function defined in Section 5 of [RFC5246], where the HMAC_GOSTR3411_2012_256 function (defined in Section 4.1.1 of this document) is used as the HMAC_hash function.

\[
\text{PRF_TLS_GOSTR3411_2012_256 (secret, label, seed)} = \\
\text{P_GOSTR3411_2012_256 (secret, label | seed)}.
\]

Label and seed values MUST be assigned by a protocol, their lengths SHOULD be fixed by a protocol in order to avoid possible collisions.

4.2.1.2. PRF_TLS_GOSTR3411_2012_512

This is the transformation providing the pseudorandom function for the TLS protocol (1.0 and higher versions) in accordance with GOST R 34.11-2012 [GOST3411-2012]. It uses the P_GOSTR3411_2012_512 function that is similar to the P_hash function defined in Section 5 of [RFC5246], where the HMAC_GOSTR3411_2012_512 function (defined in Section 4.1.2 of this document) is used as the HMAC_hash function.

\[
\text{PRF_TLS_GOSTR3411_2012_512 (secret, label, seed)} = \\
\text{P_GOSTR3411_2012_512 (secret, label | seed)}.
\]

Label and seed values MUST be assigned by a protocol, their lengths SHOULD be fixed by a protocol in order to avoid possible collisions.

4.2.2. PRFs for the IKEv2 Protocol Based on GOST R 34.11-2012

The specification for the Internet Key Exchange protocol version 2 (IKEv2) [RFC7296] defines the usage of PRFs in various parts of the protocol for the purposes of generating and authenticating keying material.

IKEv2 has no default PRF. This document specifies that HMAC_GOSTR3411_2012_256 may be used as the "prf" function in the "prf+" function for the IKEv2 protocol.
4.3. VKO Algorithms for Key Agreement

This section specifies the key agreement algorithms based on GOST R 34.10-2012 [GOST3410-2012].

4.3.1. VKO_GOSTR3410_2012_256

The VKO_GOSTR3410_2012_256 transformation is used for agreement of 256-bit keys and is based on the 256-bit version of GOST R 34.11-2012 [GOST3411-2012]. This algorithm can be applied for a key agreement using GOST R 34.10-2012 [GOST3410-2012] with 256-bit or 512-bit private keys.

The algorithm is designed to produce an encryption key or a keying material of size 256 bits to be used in various cryptographic protocols. A key or a keying material KEK_VKO (x, y, UKM) is produced from the private key x of one side, the public key y*P of the opposite side and the User Keying Material (UKM) value.

The algorithm can be used for static and ephemeral keys with the public key size n >= 512 bits including the case where one side uses a static key and the other uses an ephemeral one.

The UKM parameter is optional (the default UKM = 1) and can take any integer value from 1 to 2^(n/2)-1. It is allowed to use a non-zero UKM of an arbitrary size that does not exceed n/2 bits. If at least one of the parties uses static keys, the RECOMMENDED length of UKM is 64 bits or more.

KEK_VKO (x, y, UKM) is calculated using the formulas:

KEK_VKO (x, y, UKM) = H_256 (K (x, y, UKM)),

K (x, y, UKM) = (m/q*UKM*x mod q)*(y*P),

where m and q are the parameters of an elliptic curve defined in the GOST R 34.10-2012 [GOST3411-2012] standard (m is an elliptic curve points group order, q is an order of a cyclic subgroup), P is a non-zero point of the subgroup; P is defined by a protocol.

This algorithm is defined similar to the one specified in Section 5.2 of [RFC4357], but applies the hash function H_256 instead of the hash function GOST R 34.11-94 [GOST3411-94] (referred to as "gostR3411").
In addition, \( K(x, y, UKM) \) is calculated with public key size \( n \geq 512 \) bits and \( UKM \) has a size up to \( n/2 \) bits.

### 4.3.2. VKO_GOSTR3410_2012_512

The VKO_GOSTR3410_2012_512 transformation is used for agreement of 512-bit keys and is based on the 512-bit version of GOST R 34.11-2012 [GOST3411-2012]. This algorithm can be applied for a key agreement using GOST R 34.10-2012 [GOST3410-2012] with 512-bit private keys.

The algorithm is designed to produce an encryption key or a keying material of size 512 bits to be used in various cryptographic protocols. A key or a keying material \( KEK_{VKO} (x, y, UKM) \) is produced from the private key \( x \) of one side, the public key \( y*P \) of the opposite side and the \( UKM \) value, considered as an integer.

The algorithm can be used for static and ephemeral keys with the public key size \( n \geq 1024 \) bits including the case where one side uses a static key and the other uses an ephemeral one.

The \( UKM \) parameter is optional (the default \( UKM = 1 \)) and can take any integer value from 1 to \( 2^{n/2} - 1 \). It is allowed to use a non-zero \( UKM \) of an arbitrary size that does not exceed \( n/2 \) bits. If at least one of the parties uses static keys, the RECOMMENDED length of \( UKM \) is 128 bits or more.

\( KEK_{VKO} (x, y, UKM) \) is calculated using the formulas:

\[
KEK_{VKO} (x, y, UKM) = H_{512} (K (x, y, UKM)),
\]

\[
K (x, y, UKM) = (m/q*UKM*x mod q) \ast (y*P),
\]

where \( m \) and \( q \) are the parameters of an elliptic curve defined in the GOST R 34.10-2012 [GOST3411-2012] standard (\( m \) is an elliptic curve points group order, \( q \) is an order of a cyclic subgroup), \( P \) is a non-zero point of the subgroup; \( P \) is defined by a protocol.

This algorithm is defined similar to the one specified in Section 5.2 of [RFC4357], but applies the hash function \( H_{512} \) instead of the hash function GOST R 34.11-94 [GOST3411-94] (referred to as "gostR3411"). In addition, \( K(x, y, UKM) \) is calculated with public key size \( n \geq 1024 \) bits and \( UKM \) has a size up to \( n/2 \) bits.
4.4. The Key Derivation Function KDF_TREE_GOSTR3411_2012_256

The key derivation function KDF_TREE_GOSTR3411_2012_256 based on the HMAC_GOSTR3411_2012_256 function is given by:

\[
\text{KDF_TREE_GOSTR3411_2012_256} (K_{in}, \text{label}, \text{seed}, R) = K(1) | K(2) | K(3) | K(4) | \ldots ,
\]

\[
K(i) = \text{HMAC_GOSTR3411_2012_256} (K_{in}, [i]_b | \text{label} | 0x00 | \text{seed} | [L]_b), i \geq 1,
\]

where:

- \( K_{in} \) Derivation key.
- \( \text{label}, \text{seed} \) The parameters that MUST be assigned by a protocol; their lengths SHOULD be fixed by a protocol.
- \( R \) A fixed external parameter, with possible values of 1, 2, 3, or 4.
- \( i \) Iteration counter.
- \([i]_b\) Byte representation of the iteration counter (in the network byte order); the number of bytes in the representation \([i]_b\) is equal to \( R \) (no more than 4 bytes).
- \( L \) The required size (in bits) of the generated keying material (an integer, not exceeding \(256 \times (2^{8 \times R} - 1)\)).
- \([L]_b\) Byte representation of \( L \), in network byte order (variable length: no leading zero bytes added).

The key derivation function KDF_TREE_GOSTR3411_2012_256 is intended for generating a keying material of size \( L \), not exceeding \(256 \times (2^{8 \times R} - 1)\) bits, and utilizing general principles of the input and output for the key derivation function outlined in Section 5.1 of NIST SP 800-108 [NISTSP800-108]. The HMAC_GOSTR3411_2012_256 algorithm described in Section 4.1.1 is selected as a pseudorandom function.

Each key derived from the keying material formed using the derivation key \( K_{in} \) (0-level key) may be a 1-level derivation key and may be used to generate a new keying material. The keying material derived from the first level derivation key can be split down into the second level derivation keys. The application of this procedure leads to the construction of the key tree with the root key and the formation
of the keying material to the hierarchy of the levels, as described in Section 6 of NIST SP 800-108 [NISTSP800-108]. The partitioning procedure for keying material at each level is defined in accordance with a specific protocol.

4.5. The Key Derivation Function KDF_GOSTR3411_2012_256

The KDF_GOSTR3411_2012_256 function is equivalent to the function KDF_TREE_GOSTR3411_2012_256, when R = 1, L = 256, and is given by:

\[
\text{KDF_GOSTR3411_2012_256} (K_{\text{in}}, \text{label}, \text{seed}) = \text{HMAC_GOSTR3411_2012_256} (K_{\text{in}}, 0x01 | \text{label} | 0x00 | \text{seed} | 0x01 | 0x00),
\]

where:

- \( K_{\text{in}} \) Derivation key.
- \( \text{label}, \text{seed} \) The parameters that MUST be assigned by a protocol; their lengths SHOULD be fixed by a protocol.

4.6. Key Wrap and Key Unwrap

Wrapped representation of a secret key \( K \) (256-bit GOST 28147-89 [GOST28147-89] key, 256-bit or 512-bit GOST R 34.10-2012 [GOST3410-2012] private key) is formed as follows by using a given export key \( K_e \) (GOST 28147-89 [GOST28147-89] key) and a random seed vector:

1. Generate a random seed vector from 8 up to 16 bytes.
2. With the key derivation function, using an export key \( K_e \) as a derivation key, produce a key KEK_e (\( K_e, \text{seed} \)), where:

   \[
   \text{KEK}_e (K_e, \text{seed}) = \text{KDF_GOSTR3411_2012_256} (K_e, \text{label}, \text{seed}),
   \]
   
   where the KDF_GOSTR3411_2012_256 function (see Section 4.5) is used as a key derivation function for the fixed label value

   \[
   \text{label} = (0x26 | 0xBD | 0xB8 | 0x78).
   \]
3. GOST 28147-89 [GOST28147-89] Message Authentication Code (MAC) value (4-byte) for the data \( K \) and the key KEK_e (\( K_e, \text{seed} \)) is calculated; the initialization vector (IV) in this case is equal to the first 8 bytes of seed. The resulting value is denoted as CEK_MAC.
4. The key K is encrypted with the GOST 28147-89 [GOST28147-89] algorithm in the Electronic Codebook (ECB) mode with the key KEK_e (K_e, seed). The result is denoted as CEK_ENC.

5. The wrapped representation of the key is (seed | CEK_ENC | CEK_MAC).

The value of key K is restored from the wrapped representation of the key and the export key K_e as follows:

1. Obtain the seed, CEK_ENC and CEK_MAC values from the wrapped representation of the key.

2. With the key derivation function, using the export key K_e as a derivation key, produce a key KEK_e(K_e, seed), where:

\[ KEK_e (K_e, seed) = KDF_{GOSTR3411\_2012\_256} (K_e, label, seed), \]

where the KDF_{GOSTR3411\_2012\_256} function (see Section 4.5) is used as a key derivation function for the fixed label value

\[ label = (0x26 | 0xBD | 0xB8 | 0x78). \]

3. The CEK_ENC field is decrypted with the GOST 28147-89 [GOST28147-89] algorithm in the Electronic Codebook (ECB) mode with the key KEK_e(K_e, seed). The unwrapped key K is assumed to be equal to the result of decryption.

4. GOST 28147-89 [GOST28147-89] MAC value (4-byte) for the data K and the key KEK_e(K_e, seed) is calculated; the initialization vector (IV) in this case is equal to the first 8 bytes of seed. If the result is not equal to CEK_MAC, an error is returned.

The GOST 28147-89 [GOST28147-89] algorithm is used with the parameter set defined in Appendix C of this document.

5. The Parameters of Elliptic Curves

This section defines the elliptic curves parameters and object identifiers that are RECOMMENDED for usage with the signature and verification algorithms of the digital signature in accordance with the GOST R 34.10-2012 [GOST3410-2012] standard and with the key agreement algorithms VKO_GOSTR3410_2012_256 and VKO_GOSTR3410_2012_512.

This document does not negate the use of other parameters of elliptic curves.
5.1. Canonical Form

This section defines the elliptic curves parameters of the GOST R 34.10-2012 [GOST3410-2012] standard for the case of elliptic curves with prime 512-bit moduli in canonical (short Weierstrass) form, that is given by the following equation defined in GOST R 34.10-2012 [GOST3410-2012]:

$$y^2 = x^3 + ax + b \pmod{p}.$$  

In case of elliptic curves with 256-bit prime moduli, the parameters defined in [RFC4357] are proposed for use.

5.1.1. Parameters and Object Identifiers

The parameters for each elliptic curve are represented by the following values, which are defined in GOST R 34.10-2012 [GOST3410-2012]:

- $p$: the characteristic of the underlying prime field;
- $a$, $b$: the coefficients of the equation of the elliptic curve in the canonical form;
- $m$: the elliptic curve group order;
- $q$: the elliptic curve subgroup order;
- $(x, y)$: the coordinates of the point $P$ (generator of the subgroup of order $q$) of the elliptic curve in the canonical form.

Both sets of the parameters are presented as structures of the form:

```plaintext
SEQUENCE {
  p    INTEGER,
  a    INTEGER,
  b    INTEGER,
  m    INTEGER,
  q    INTEGER,
  x    INTEGER,
  y    INTEGER
}
```

The parameter sets have the following object identifiers:

1. id-tc26-gost-3410-12-512-paramSetA ::= 
   (iso(1) member-body(2) ru(643) rosstandart(7) tc26(1) constants(2) sign-constants(1) gost-3410-12-512-constants(2) paramSetA(1));
2. \text{id-tc26-gost-3410-12-512-paramSetB}::= \{iso(1) member-body(2)
    ru(643) rosstandart(7) tc26(1) constants(2) sign-constants(1)
    gost-3410-12-512-constants(2) paramSetB(2)\}.

The corresponding values of the parameter sets can be found in
Appendix A.1.

5.2. Twisted Edwards Form

This section defines the elliptic curves parameters and object
identifiers of the GOST R 34.10-2012 [GOST3410-2012] standard for the
case of elliptic curves that have a representation in the twisted
Edwards form with prime 256-bit and 512-bit moduli.

A twisted Edwards curve $E$ over a finite prime field $F_p$, $p > 3$, is an
equivalent representation in the twisted Edwards form with prime 256-bit and 512-bit moduli.

A twisted Edwards curve has an equivalent representation in the short
Weierstrass form defined by parameters $a$, $b$. The parameters $a$, $b$, $e$, and $d$ are related as follows:

\begin{align*}
    a &= s^2 - 3t^2 \pmod{p}, \\
    b &= 2t^3 - t^2s^2 \pmod{p},
\end{align*}

where:

\begin{align*}
    s &= (e - d)/4 \pmod{p}, \\
    t &= (e + d)/6 \pmod{p}.
\end{align*}

Coordinate transformations are defined as follows:

\begin{align*}
    (u, v) \rightarrow (x, y) &= (s(1 + v)/(1 - v) + t, s(1 + v)/(1 - v)u), \\
    (x, y) \rightarrow (u, v) &= ((x - t)/y, (x - t - s)/(x - t + s)).
\end{align*}

5.2.1. Parameters and Object Identifiers

The parameters for each elliptic curve are represented by the
following values, which are defined in GOST R 34.10-2012
[GOST3410-2012]:

\begin{itemize}
    \item $p$ The characteristic of the underlying prime field.
    \item $a$, $b$ The coefficients of the equation of the elliptic curve in the
          canonical form.
\end{itemize}
e, d The coefficients of the equation of the elliptic curve in the
twisted Edwards form.

m The elliptic curve group order.

q The elliptic curve subgroup order.

(x, y) The coordinates of the point P (generator of the subgroup of
order q) of the elliptic curve in the canonical form.

(u, v) The coordinates of the point P (generator of the subgroup of
order q) of the elliptic curve in the twisted Edwards form.

Both sets of the parameters are presented as ASN structures of the
form:

```plaintext
SEQUENCE {
    p       INTEGER,
    a       INTEGER,
    b       INTEGER,
    e       INTEGER,
    d       INTEGER,
    m       INTEGER,
    q       INTEGER,
    x       INTEGER,
    y       INTEGER,
    u       INTEGER,
    v       INTEGER
}
```

The parameter sets have the following object identifiers:

1. id-tc26-gost-3410-2012-256-paramSetA ::= {iso(1) member-body(2)
    ru(643) rosstandart(7) tc26(1) constants(2) sign-constants(1)
    gost-3410-12-256-constants(1) paramSetA(1)};

2. id-tc26-gost-3410-2012-512-paramSetC ::= {iso(1) member-body(2)
    ru(643) rosstandart(7) tc26(1) constants(2) sign-constants(1)
    gost-3410-12-512-constants(2) paramSetC(3)}.

The corresponding values of the parameter sets can be found in
Appendix A.2.

6. Security Considerations

This entire document is about security considerations.
7. References

7.1. Normative References


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7.2. Informative References


Appendix A. Values of the Parameter Sets

A.1. Canonical Form Parameters

Parameter set: id-tc26-gost-3410-12-512-paramSetA

SEQUENCE

{  
  OBJECT IDENTIFIER
  id-tc26-gost-3410-12-512-paramSetA
  SEQUENCE

  {  
    INTEGER
      00 FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF
      FF FF FF FF FF FF FF FF FF FF FF FF FF FF FD C7
    INTEGER
      00 FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF
      FF FF FF FF FF FF FF FF FF FF FF FF FF FF FD C4
    INTEGER
      00 E8 C2 50 5D ED FC 86 DD C1 BD 0B 2B 66 67 F1
      DA 34 B8 25 74 76 1C B0 E8 79 BD 08 1C FD 0B 62
      65 EE 3C B0 90 F3 0D 27 61 4C B4 57 40 10 DA 90
      DD 86 2E F9 D4 EB EE 47 61 50 31 90 78 5A 71 C7
      60
    INTEGER
      00 FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF
      FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF
      FF 27 E6 95 32 F4 8D 89 11 6F F2 2B 8D 4E 05 60
      60 9B 4B 38 AB FA D2 B8 5D CA CD B1 41 1F 10 B2
      75
    INTEGER
      00 FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF
      FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF
      FF 27 E6 95 32 F4 8D 89 11 6F F2 2B 8D 4E 05 60
      60 9B 4B 38 AB FA D2 B8 5D CA CD B1 41 1F 10 B2
      75
    INTEGER
      03
  }

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INTEGER
    75 03 CF E8 7A 83 6A E3 A6 1B 88 16 E2 54 50 E6
CE 5E 1C 93 AC F1 AB C1 77 80 64 FD CB EF A9 21
DF 16 26 BE 4F D0 36 E9 3D 75 E6 A5 0E 3A 41 E9
80 28 FE 5F C2 35 F5 B8 89 A5 89 CB 52 15 F2 A4
}
)

Parameter set: id-tc26-gost-3410-12-512-paramSetB

SEQUENCE
{
 OBJECT IDENTIFIER
     id-tc26-gost-3410-12-512-paramSetB
 SEQUENCE
{
     INTEGER
         00 80 00 00 00 00 00 00 00 00 00 00 00 00 00 00
         00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
         00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
         00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
         6F
     INTEGER
         00 80 00 00 00 00 00 00 00 00 00 00 00 00 00 00
         00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
         00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
         00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
         6C
     INTEGER
         68 7D 1B 45 9D C8 41 45 7E 3E 06 CF 6F 5E 25 17
         B9 7C 7D 61 4A F1 38 BC BF 85 DC 80 6C 4B 28 9F
         3E 96 5D 2D B1 41 6D 21 7F 8B 27 6F AD 1A B6 9C
         50 F7 8B EE 1F A3 10 6E FB 8C CB C7 C5 14 01 16
     INTEGER
         00 80 00 00 00 00 00 00 00 00 00 00 00 00 00 00
         00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
         01 49 A1 EC 14 25 65 A5 45 AC FD B7 7B D9 D4 0C
         FA 8B 99 67 12 10 1B EA 0E C6 34 6C 54 37 4F 25
         BD
     INTEGER
         00 80 00 00 00 00 00 00 00 00 00 00 00 00 00 00
         00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
         01 49 A1 EC 14 25 65 A5 45 AC FD B7 7B D9 D4 0C
         FA 8B 99 67 12 10 1B EA 0E C6 34 6C 54 37 4F 25
         BD
     INTEGER
         02

A.2. Twisted Edwards Form Parameters

Parameter set: id-tc26-gost-3410-2012-256-paramSetA

SEQUENCE
{
  OBJECT IDENTIFIER
  id-tc26-gost-3410-2012-256-paramSetA
  SEQUENCE
  {
    INTEGER
    00 FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF
    FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF FD
    97
    INTEGER
    00 C2 17 3F 15 13 98 16 73 AF 48 92 C2 30 35 A2
    7C E2 5E 20 13 BF 95 AA 33 B2 2C 65 6F 27 7E 73
    35
    INTEGER
    29 5F 9B AE 74 28 ED 9C CC 20 E7 C3 59 A9 D4 1A
    22 FC CD 91 08 E1 7B F7 BA 93 37 A6 F8 AE 95 13
    INTEGER
    01
    INTEGER
    06 05 F6 B7 C1 83 FA 81 57 8B C3 9C FA D5 18 13
    2B 9D F6 28 97 00 9A F7 E5 22 C3 2D 6D C7 BF FB
    INTEGER
    01 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
    00 3F 63 37 7F 21 ED 98 D7 04 56 BD 55 B0 D8 31
    9C
    INTEGER
    40 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
    0F D8 CD DF C8 7B 66 35 C1 15 AF 55 6C 36 0C 67
    INTEGER
    00 91 E3 84 43 A5 E8 2C 0D 88 09 23 42 57 12 B2
    BB 65 8B 91 96 93 2E 02 C7 8B 25 82 FE 74 2D AA
    28
  }
}
Parameter set: id-tc26-gost-3410-2012-512-paramSetC

SEQUENCE
{
  OBJECT IDENTIFIER
  id-tc26-gost-3410-2012-512-paramSetC
  SEQUENCE
  {
    INTEGER
    00 FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF
    FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF
    FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF
    FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF
    FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF
    FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF
    FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF
    FF FF FF FF FF FF FF FF FF FF FF FF FF FF FF FD
    C7
    INTEGER
    00 DC 92 03 E5 14 A7 21 87 54 85 A5 29 D2 C7 22
    FB 18 7B C8 98 0E B8 66 64 4D E4 1C 68 E1 43 06
    45 46 E8 61 C0 E2 C9 ED D9 2A DE 71 F4 6F CF 50
    FF 2A D9 7F 95 1F DA 9F 2A 2E B6 54 6F 39 68 9B
    D3
    INTEGER
    00 B4 C4 EE 28 CE BC 6C 2C 8A C1 29 52 CF 37 F1
    6A C7 EF B6 A9 F6 9F 4B 57 FF DA 2E 4F 0D E5 AD
    E0 38 CB C2 FF F7 19 D2 C1 8D E0 28 4B 8B FE F3
    B5 2B 8C C7 A5 F5 BF 0A 3C 8D 23 19 A5 31 25 57
    E1
    INTEGER
    01
    INTEGER
    00 9E 4F 5D 8C 01 7D 8D 9F 13 A5 CF 3C DF 5B FE
    4D AB 40 2D 54 19 8E 31 EB DE 28 A0 62 10 50 43
    9C A6 B3 9E 0A 51 5C 06 B3 04 E2 CE 43 E7 9E 36
    9E 91 A0 CF C2 BC 2A 22 B4 CA 30 2D BB 33 EE 75 50
  }
}
Appendix B.  Test Examples

1) HMAC_GOSTR3411_2012_256

Key K:

00 01 02 03 04 05 06 07 08 09 0a 0b 0c 0d 0e 0f
10 11 12 13 14 15 16 17 18 19 1a 1b 1c 1d 1e 1f

T:

01 26 bd b8 78 00 4f 21 43 41 45 65 63 78 01 00

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HMAC_GOSTR3411_2012_256 (K, T) value:

a1 aa 5f 7d e4 02 d7 b3 d3 23 f2 99 1c 8d 45 34
01 31 37 01 0a 83 75 4f d0 af 6d 7c d4 92 2e d9

2) HMAC_GOSTR3411_2012_512

Key K:

00 01 02 03 04 05 06 07 08 09 0a 0b 0c 0d 0e 0f
10 11 12 13 14 15 16 17 18 19 1a 1b 1c 1d 1e 1f

T:

01 26 bd b8 78 00 af 21 43 41 45 65 63 78 01 00

HMAC_GOSTR3411_2012_512 (K, T) value:

a5 9b ab 22 ec ae 19 c6 5f bd e6 e5 f4 e9 f5 d8
54 9d 31 f0 37 f9 df 9b 90 55 00 e1 71 92 3a 77
3d 5f 15 30 f2 ed 7e 96 4c b2 ee dc 29 e9 ad 2f
3a fe 93 b2 81 4f 79 f5 00 0f fc 03 66 c2 51 e6

3) PRF_TLS_GOSTR3411_2012_256

Key K:

00 01 02 03 04 05 06 07 08 09 0a 0b 0c 0d 0e 0f
10 11 12 13 14 15 16 17 18 19 1a 1b 1c 1d 1e 1f

Seed:

18 47 1d 62 2d c6 55 c4 d2 d2 26 96 91 ca 4a 56
0b 50 ab a6 63 55 3a f2 41 f1 ad a8 82 c9 f2 9a

Label:

11 22 33 44 55

Output T1:

ff 09 66 4a 44 74 58 65 94 4f 83 9e bb 48 96 5f
15 44 ff 1c c8 e8 f1 6f 24 7e e5 f8 a9 eb e9 7f
Output T2:

c4 e3 c7 90 0e 46 ca d3 db 6a 01 64 30 63 04 0e
c6 7f c0 fd 5c d9 f9 04 65 23 52 37 bd ff 2c 02

4) PRF_TLS_GOST3411_2012_512

Key K:

00 01 02 03 04 05 06 07 08 09 0a 0b 0c 0d 0e 0f
10 11 12 13 14 15 16 17 18 19 1a 1b 1c 1d 1e 1f

Seed:

18 47 1d 62 2d c6 55 c4 d2 d2 26 96 91 ca 4a 56
0b 50 ab a6 63 55 3a f2 41 f1 ad a8 82 c9 f2 9a

Label:

11 22 33 44 55

Output T1:

f3 51 87 a3 dc 96 55 11 3a 0e 84 d0 6f d7 52 6c
5f c1 fb de c1 a0 e4 67 3d d6 d7 9d 0b 92 0e 65
ad 1b c4 7b b0 83 b3 85 1c b7 cd 8e 7e 6a 91 la
62 6c f0 2b 29 e9 e4 a5 8e d7 66 a4 49 a7 29 6d

Output T2:

e6 1a 7a 26 c4 d1 ca ee cf d8 0c ca 65 c7 1f 0f
88 c1 f8 22 c0 e8 c0 ad 94 9d 03 fe e1 39 57 9f
72 ba 0c 3d 32 c5 f9 54 f1 cc cd 54 08 1f c7 44
02 78 cb a1 fe 7b 7a 17 a9 86 fd ff 5b d1 5d 1f

5) PRF_IPSEC_PRFPLUS_GOST3411_2012_256

Key K:

c9 a9 a7 73 20 e2 cc 55 9e d7 2d ce 6f 47 e2 19
2c ce a9 5f a6 48 67 05 82 c0 54 c0 ef 36 c2 21

Data S:

01 26 bd b8 78 00 1d 80 60 3c 85 44 c7 27 01 00
Output T1:

2d e5 ee 84 e1 3d 7b e5 36 16 67 39 13 37 0a b0
54 c0 74 b7 9b 69 a8 a8 46 82 a9 f0 4f ec d5 87

Output T2:

29 f6 0d da 45 7b f2 19 aa 2e f9 5d 7a 59 be 95
4d e0 08 f4 a5 0d 50 4d bd b6 90 be 68 06 01 53

6) PRF_IPSEC_PRFPLUS_GOSTR3411_2012_512

Key K:
c9 a9 a7 73 20 e2 cc 55 9e d7 2d ce 6f 47 e2 19
2c ce a9 5f a6 48 67 05 82 c0 54 c0 ef 36 c2 21

Data S:

01 26 bd b8 78 00 1d 80 60 3c 85 44 c7 27 01 00

Output T1:

5d a6 71 43 a5 f1 2a 6d 6e 47 42 59 6f 39 24 3f
cc 61 57 45 91 5b 32 59 10 06 ff 78 a2 08 63 d5
f8 8e 4a fc 17 fb be 70 b9 50 95 73 db 00 5e 96
26 36 98 46 cb 86 19 99 71 6c 16 5d d0 6a 15 85

Output T2:

48 34 49 5a 43 74 6c b5 3f 0a ba 3b c4 6e bc f8
77 3c a6 4a d3 43 c1 22 ee 2a 57 75 57 03 81 57
ee 9c 38 8d 96 ef 71 d5 8b e5 c1 ef a1 af a9 9e
be 83 e3 9d 00 e1 9a 5d 03 dc d6 0a 01 bc a8 e3

7) VKO_GOSTR3410_2012_256 with 256-bit output on the GOST R 34.10-2012 512-bit keys with id-tc26-gost-3410-12-512-paramSetA

UKM value:

1d 80 60 3c 85 44 c7 27

Private key x of A:

c9 90 ec d9 72 fc e8 4e c4 db 02 27 78 f5 0f ca
c7 26 f4 67 08 38 4b 8d 45 83 04 96 2d 71 47 f8
c2 db 41 ce f2 2c 90 b1 02 f2 96 84 04 f9 b9 be
6d 47 c7 96 92 d8 18 26 b3 2b 8d ac a4 3c b6 67
Public key x*P of A (curve point (X, Y)):

aa b0 ed a4 ab ff 21 20 8d 18 79 9f b9 a8 55 66
54 ba 78 70 eb a1 0c b9 ab b2 53 ec 56 dc f5
d3 cc ba 61 92 e4 64 e6 e5 bc b6 de a1 37 79 2f
24 31 f6 c8 97 eb 1b 3c 0c c1 43 27 b1 ad c0 a7
91 46 13 a3 07 4e 36 3a ed b2 04 d3 8d 35 63 97
1b d8 75 8e 87 8c 9d b1 14 03 72 1b 48 00 2d 38
46 1f 92 47 2d 40 ea 92 f9 95 8c 0f fa 4c 93 75
64 01 b9 7f 89 fd be 0b 5e 46 e4 a4 63 1c db 5a

Private key y of part B:

48 c8 59 f7 b6 f1 15 85 88 7c c0 5e c6 ef 13 90
cf ea 73 9b 1a 18 c0 d4 66 22 93 ef 63 b7 9e 3b
80 14 07 0b 44 91 85 90 b4 b9 96 ac fe a4 ed fb
bb cc cc 8c 06 ed d8 bf 5b da 92 a5 13 92 d0 db

Public key y*P of B (curve point (X, Y)):

19 2f e1 83 b9 71 3a 07 72 53 c7 2c 87 35 de 2e
a4 2a 3d bc 66 ea 31 78 38 b6 5f a3 25 23 cd 5e
fc a9 74 ed a7 c8 63 f4 95 4d 11 47 f1 f2 b2 5c
39 5f ce 1c 12 91 75 e8 76 d1 32 e9 4e d5 a6 51
04 88 3b 41 4c 9b 59 2e c4 dc 84 82 6f 07 d0 b6
d9 00 6d da 17 6c e4 8c 39 1e 3f 97 d1 02 e0 3b
b5 98 bf 13 2a 22 8a 45 f7 20 1a ba 08 fc 52 4a
2d 77 e4 3a 36 2a b0 22 ad 40 28 f7 5b de 3b 79

KEK_VKO value:

c9 a9 a7 73 20 e2 cc 55 9e d7 2d ce 6f 47 e2 19
2c ce a9 5f a6 48 67 05 82 c0 54 c0 ef 36 c2 21

8) VKO_GOSTR3410_2012_512 with 512-bit output on the GOST
R 34.10-2012 512-bit keys with id-tc26-gost-3410-12-512-paramSetA

UKM value:

1d 80 60 3c 85 44 c7 27

Private key x of A:

c9 90 ec d9 72 fc e8 4e c4 db 02 27 78 f5 0f ca
c7 26 f4 67 08 38 4b 8d 45 83 04 96 2d 71 47 f8
c2 db 41 ce f2 2c 90 b1 02 f2 96 84 04 f9 b9 be
6d 47 c7 96 92 d8 18 26 b3 2b 8d ac a4 3c b6 67
Public key $x^*P$ of A (curve point $(X, Y)$):

```
aa b0 ed a4 ab ff 21 20 8d 18 79 9f b9 a8 55 66
54 ba 70 30 70 eb a1 0c b9 ab b2 53 ec 56 dc f5
d3 cc ba 61 92 e4 64 e6 e5 bc b6 de a1 37 79 2f
24 31 f6 c8 97 eb 1b 3c 0c c1 43 27 b1 ad c0 a7
91 46 13 a3 07 4e 36 3a ed b2 04 d3 8d 35 63 97
1b d8 75 8e 87 8c 9d b1 14 03 72 1b 48 00 2d 38
46 1f 92 47 2d 40 ea 92 f9 95 8c 0f fa 4c 93 75
64 01 b9 7f 89 fd be 0b 5e 46 e4 a4 63 1c db 5a
```

Private key $y$ of B:

```
48 c8 59 f7 b6 f1 15 85 88 7c c0 5e c6 ef 13 90
cf ea 73 9b 1a 18 c0 d4 66 22 93 ef 63 b7 9e 3b
80 14 07 0b 44 91 85 90 b4 b9 96 ac fe a4 ed fb
bb cc cc 8c 06 ed d8 bf 5b da 92 a5 13 92 d0 db
```

Public key $y^*P$ of B (curve point $(X, Y)$):

```
19 2f e1 83 b9 71 3a 07 72 53 c7 2c 87 35 de 2e
a4 2a 3d bc 66 ea 31 78 38 b6 5f a3 25 23 cd 5e
fc a9 74 ed a7 c8 63 f4 95 4d 11 47 f1 f2 b2 5c
39 5f ce 1c 12 91 75 e8 76 d1 32 e9 4e d5 a6 51
04 88 3b 41 4c 9b 59 2e c4 dc 84 82 6f 07 d0 b6
d9 00 6d da 17 6c e4 8c 39 1e 3f 97 d1 02 e0 3b
b5 98 bf 13 2a 22 8a 45 f7 20 1a ba 08 fc 52 4a
2d 77 e4 3a 36 2a b0 22 ad 40 28 f7 5b de 3b 79
```

KEK_VKO value:

```
79 f0 02 a9 69 40 ce 7b de 32 59 a5 2e 01 52 97
ad aa d8 45 97 a0 d2 05 b5 0e 3e 17 19 f9 7b fa
7e e1 d2 66 1f a9 97 9a 5a a2 35 b5 58 a7 e6 d9
f8 8f 98 2d d6 3f c3 5a 8e c0 dd 5e 24 2d 3b df
```

9) Key derivation function KDF_GOSTR3411_2012_256

K_in key:

```
00 01 02 03 04 05 06 07 08 09 0a 0b 0c 0d 0e 0f
10 11 12 13 14 15 16 17 18 19 1a 1b 1c 1d 1e 1f
```

Label:

```
26 bd b8 78
```
Seed:
af 21 43 41 45 65 63 78

KDF(K_in, label, seed) value:

a1 aa 5f 7d e4 02 d7 b3 d3 23 f2 99 1c 8d 45 34
01 31 37 01 0a 83 75 4f d0 af 6d 7c d4 92 2e d9

10) Key derivation function KDF_TREE_GOSTR3411_2012_256

Output size of L:
512

K_in key:
00 01 02 03 04 05 06 07 08 09 0a 0b 0c 0d 0e 0f
10 11 12 13 14 15 16 17 18 19 1a 1b 1c 1d 1e 1f

Label:
26 bd b8 78

Seed:
af 21 43 41 45 65 63 78

K1:
22 b6 83 78 45 c6 be f6 5e a7 16 72 b2 65 83 10
86 d3 c7 6a eb e6 da e9 1c ad 51 d8 3f 79 d1 6b

K2:
07 4c 93 30 59 9d 7f 8d 71 2f ca 54 39 2f 4d dd
e9 37 51 20 6b 35 84 c8 f4 3f 9e 6d c5 15 31 f9

R:
1
11) Key wrap and unwrap with the szOID_Gost28147_89_TC26_Z_ParamSet parameters

Key $K_e$:
00 01 02 03 04 05 06 07 08 09 0a 0b 0c 0d 0e 0f
10 11 12 13 14 15 16 17 18 19 1a 1b 1c 1d 1e 1f

Key $K$:
20 21 22 23 24 25 26 27 28 29 2a 2b 2c 2d 2e 2f
30 31 32 33 34 35 36 37 38 39 3a 3b 3c 3d 3e 3f

Seed:
af 21 43 41 45 65 63 78

Label:
26 bd b8 78

$KEK_e$(seed) = KDF_GOSTR3411_2012_256($K_e$, label, seed):
a1 aa 5f 7d e4 02 d7 b3 d3 23 f2 99 1c 8d 45 34
01 31 37 01 0a 83 75 4f d0 af 6d 7c d4 92 2e d9

$CEK_{MAC}$:
be 33 f0 52

$CEK_{ENC}$:
d1 55 47 f8 ee 85 12 1b c8 7d 4b 10 27 d2 60 27
dc c0 71 bb a6 e7 2f 3f ec 6f 62 0f 56 83 4c 5a
Appendix C. GOST 28147-89 Parameter Set

The parameter set has the following object identifier:

\[
\text{id-tc26-gost-28147-param-Z::} &= \{\text{iso(1) member-body(2) ru(643)} \\
& \text{rosstandart(7) tc26(1) constants(2) cipher-constants(5)} \\
& \text{gost-28147-constants(1) param-Z(1)}\}
\]

The parameter set is defined below:

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<thead>
<tr>
<th>x</th>
<th>K1(x)</th>
<th>K2(x)</th>
<th>K3(x)</th>
<th>K4(x)</th>
<th>K5(x)</th>
<th>K6(x)</th>
<th>K7(x)</th>
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<td>b</td>
<td>c</td>
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